

SEMICLASSICAL THEORY OF h/e AHARONOV-BOHM OSCILLATION IN BALLISTIC REGIMES

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We study the magneto-transport in Aharonov-Bohm (AB) billiards forming doubly connected structures. In these systems, non-averaged conductance oscillates as a function of magnetic flux with period h/e . We derive formulas of the correlation function $C(\Delta\phi)$ of the magneto-conductance for chaotic and regular AB billiards by use of the semiclassical theory. The different higher harmonics behaviors for $C(\Delta\phi)$ are related to the differing distribution of classical dwelling times. The AB oscillation in ballistic regimes provides an experimental probe of quantum signatures of classical chaotic and regular dynamics.

1 Introduction

Electron transport through ballistic quantum billiards is an exceedingly rich experimental system, bearing the quantum signature of chaos.¹ One of the interesting result that has emerged concerns the magneto-transport of doubly connected ballistic billiards, i.e., Aharonov-Bohm (AB) billiards.^{2,3} We have calculated the *average* conductance for these systems and showed that the self-averaging effect causes the $h/2e$ Altshuler-Aronov-Spivak (AAS) oscillation which is ascribed to interference between time-reversed coherent back-scattering classical trajectories.² Moreover we have showed that the AAS oscillation in these systems becomes an experimental probe of the quantum chaos. Another interesting phenomenon in these systems is the h/e AB oscillation for *non-averaged* conductance. The result of numerical calculations² indicated that the period of the energy averaged conductance changed from $h/2e$ to h/e , when the range of energy average ΔE is decreased. However, little is known about the effect of chaos on the h/e AB oscillation in AB billiards. In this paper, we shall calculate the correlation function $C(\Delta\phi)$ of the *non-averaged* conductance by using the semiclassical theory and show that $C(\Delta\phi)$ is qualitatively different between chaotic and regular AB billiards.⁴

2 Semiclassical Theory

In the following, we shall derive $C(\Delta\phi)$ separately for chaotic and regular AB billiards in which uniform normal magnetic field B (AB flux) penetrates only through the hollow. The transmission amplitude from a mode m on the left

to a mode n on the right for electrons at the Fermi energy is given by

$$t_{n,m} = -i\hbar\sqrt{v_n v_m} \int dy \int dy' \psi_n^*(y') \psi_m(y) G(y', y, E_F), \quad (1)$$

where $v_m(v_n)$ and $\psi_m(\psi_n)$ are the longitudinal velocity and transverse wave function for the mode m (n) at a pair of lead wires attached to the billiards. In eq. (1), G is the retarded Green's function. In order to carry out the semiclassical approximation, we replace G by the semiclassical Green function,

$$G^{sc}(y', y, E) = \frac{2\pi}{(2\pi i\hbar)^{3/2}} \sum_{s(y,y')} \sqrt{D_s} \exp \left[\frac{i}{\hbar} S_s(y', y, E) - i\frac{\pi}{2} \mu_s \right] \quad (2)$$

where S_s is the action integral along a classical path s , the pre-exponential factor is

$$D_s = \frac{m_e}{v_F \cos \theta'} \left| \left(\frac{\partial \theta}{\partial y'} \right)_y \right| \quad (3)$$

with θ and θ' the incoming and outgoing angles, respectively, and μ is the Maslov index. Substituting eq. (2) into eq. (1) and carrying out the double integrals by the saddle-point approximation, we obtain

$$t_{n,m} = -\frac{\sqrt{2\pi i\hbar}}{2W} \sum_{s(\bar{n}, \bar{m})} \text{sgn}(\bar{n}) \text{sgn}(\bar{m}) \sqrt{\tilde{D}_s} \exp \left[\frac{i}{\hbar} \tilde{S}_s(\bar{n}, \bar{m}; E) - i\frac{\pi}{2} \tilde{\mu}_s \right], \quad (4)$$

where W is the width of the hard-wall leads and $\bar{m} = \pm m$. In eq. (4), $\tilde{S}_s(\bar{n}, \bar{m}; E) = S_s(y'_0, y_0; E) + \hbar\pi(\bar{m}y_0 - \bar{n}y'_0)/W$, $\tilde{D}_s = (m_e v_F \cos \theta')^{-1} |(\partial y / \partial \theta')_\theta|$ and $\tilde{\mu}_s = \mu_s + H(-(\partial \theta / \partial y')'_y) + H(-(\partial \theta' / \partial y')_\theta)$, respectively, where $\theta = \sin^{-1}(\bar{n}\pi/kW)$ and H is the Heaviside step function.

The fluctuations of the conductance $g = (e^2/\pi\hbar)T(k) = (e^2/\pi\hbar) \sum_{n,m} |t_{n,m}|^2$ are defined by their deviation from the classical value; in the absence of any symmetries,

$$\delta g \equiv g - g_{cl}. \quad (5)$$

In this equation $g_{cl} = (e^2/\pi\hbar)T_{cl}$, where T_{cl} is the classical total transmitted intensity. In order to characterize the h/e AB oscillation, we define the correlation function of the oscillation in magnetic field B by the average over B ,

$$C(\Delta B) \equiv \langle \delta g(B) \delta g(B + \Delta B) \rangle_B. \quad (6)$$

With use of the ergodic hypothesis, B averaging can be replaced by the k averaging, i.e.,

$$C(\Delta B) = \langle \delta g(k, B) \delta g(k, B + \Delta B) \rangle_k. \quad (7)$$

The semiclassical correlation function of transmission coefficients is given by

$$C(\Delta\phi) = \left(\frac{e^2}{\pi\hbar}\right)^2 \frac{1}{8} \left(\frac{\cosh\delta - 1}{\sinh\delta}\right)^2 \cos\left(2\pi\frac{\Delta\phi}{\phi_0}\right) \times \left\{1 + 2 \sum_{n=1}^{\infty} e^{-\delta n} \cos\left(2\pi n \frac{\Delta\phi}{\phi_0}\right)\right\}^2, \quad (8)$$

where $\delta = \sqrt{2T_0\gamma/\alpha}$.⁴ In deriving eq. (8) we have used the exponential dwelling time distribution, $N(T) \sim \exp(-\gamma T)$,⁵ and the Gaussian winding number distribution for fixed T ,⁶ i.e.,

$$P(w; T) = \sqrt{\frac{T_0}{2\pi\alpha T}} \exp\left(-\frac{w^2 T_0}{2\alpha T}\right), \quad (9)$$

where T_0 and α are the system-dependent constants corresponding to the dwelling time for the shortest classical winding trajectory and the variance of the distribution of w , respectively.

On the other hand, for the regular cases, we use $N(T) \sim T^{-\beta}$.⁵ Assuming as well the Gaussian distribution of $P(w; T)$, we get

$$C(\Delta\phi) = C(0) \cos\left(2\pi\frac{\Delta\phi}{\phi_0}\right) \times \left\{ \frac{1 + 2 \sum_{n=1}^{\infty} F\left(\beta - \frac{1}{2}, \beta + \frac{1}{2}; -\frac{n^2}{2\alpha}\right) \cos\left(2\pi n \frac{\Delta\phi}{\phi_0}\right)}{1 + 2 \sum_{n=1}^{\infty} F\left(\beta - \frac{1}{2}, \beta + \frac{1}{2}; -\frac{n^2}{2\alpha}\right)} \right\}^2, \quad (10)$$

where F is the hyper-geometric function of confluent type.⁴

Next we shall see the difference of $C(\Delta\phi)$ for chaotic and regular AB billiards in detail. In the chaotic AB billiard, a main contribution to the AB oscillation comes from the $n = 1$ component. On the other hand, for regular cases, the amplitude of the AB oscillation decays algebraically, i.e., $F \sim n^{-2\beta-1}$ for large n . This behavior is caused by the power law dwelling time distribution, i.e., $N(T) \sim T^{-\beta}$. Thus, in contrast to the chaotic cases, we can expect that the considerably higher harmonics contribution causes a noticeable deviation from the cosine function for $C(\Delta\phi)$. Therefore, the difference of $C(\Delta\phi)$ of these ballistic AB billiards can be attributed to the difference between chaotic and regular classical scattering dynamics.

3 Summary

We have investigated that magneto-transport in single ballistic billiards whose structures form AB geometry by use of semiclassical methods with a particular emphasis on the derivation of the semiclassical formulas. The existence of the AB oscillation of *non-averaged* magneto-conductance is predicted for single chaotic and regular AB billiards. Furthermore, we find that the difference between classical dynamics leads to qualitatively different behaviors for the correlation function. The AB oscillation in the ballistic regime will provide a new experimental testing ground for exploring quantum chaos.

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